PHYSICS 110A : MECHANICS 1
PROBLEM SET #1

[**1**] The one-component dynamical system $\dot{u} = f(u)$ is simple to solve graphically. Simply plot $f(u)$ *versus* u. The velocity $\dot{u}$ is then to the right in regions where $f(u) > 0$ and to the left in regions where $f(u) < 0$. Points where $f(u) = 0$, which are generically isolated, are *fixed points* of the dynamics, as discussed in §1.1.5 of the lecture notes.

Consider the modified logistic equation,

$$\frac{dN}{dt} = f(N) = rN\left(1 - \frac{N^2}{K^2}\right) \quad ,$$

with $r > 0$.

(a) Sketch $f(N)$ *versus* N (you may restrict your attention to $N \geq 0$) and draw arrows on the N axis in the direction of the flow $\dot{N}$.

(b) Identify all fixed points and classify them as either stable or unstable.

(c) Solve exactly for $N(t)$ subject to the initial condition $N(0) = N_0$ by using the method of partial fractions.

[**2**] A particle of mass m moves in the one-dimensional potential

$$U(x) = U_0 \, \frac{x^2}{a^2} \, e^{-x/a} \quad .$$

(a) Sketch $U(x)$. Identify the location(s) of any local minima and/or maxima, and be sure that your sketch shows the proper behavior as $x \to \pm\infty$.

(b) Sketch a representative set of phase curves. Identify and classify any and all fixed points. Find the energy of each and every separatrix.

(c) Sketch all the phase curves for motions with total energy $E = \frac{2}{5} U_0$. Do the same for $E = U_0$. (Recall that $e = 2.71828\ldots$.)

(d) Derive and expression for the period T of the motion when $|x| \ll a$.

[**3**] A plane flying horizontally at constant speed v_0 and at height h_0 above the sea must drop a bundle of supplies to a hapless castaway on a small raft.

(a) Write Newton's second law for the bundle as it falls from the plane, assuming gravity is the only force acting on the bundle.

(b) Solve for the bundle's position as a function of time t since the release of the bundle.

(c) How far before the raft, measured horizontally, must the pilot drop the bundle if it is to hit the raft?

(d) What is this distance if $v_0 = 50\,\mathrm{m/s}$, $h_0 = 100\,\mathrm{m}$, and $g = 9.8\,\mathrm{m/s}$?

(e) Within what interval $\pm\Delta t$ of time must the pilot release the bundle if it is to land within 10 m of the raft?

[4] A mass m is constrained to move along the x-axis subject to a velocity-dependent force $F(v) = -F_0\, e^{v/V}$, where F_0 and V are constants.

(a) Find $v(t)$ if the initial velocity is $v(0) = v_0 > 0$ at time $t = 0$.

(b) At what time does the mass come instantaneously to rest?

(c) By integrating the function $v(t)$, find $x(t)$.

(d) How far does the mass travel before it starts to turn around and reverse direction?