PHYSICS 110A : MECHANICS 1
MIDTERM EXAMINATION

[**1**] A point particle of mass m moves in one space dimension with potential energy

$$U(x) = U_0 \left(\frac{x^3}{3a^3} - \frac{x}{a} \right) \quad .$$

Here U_0 and a are both positive.

(a) What are the dimensions of a and of U_0?
`[5 points]`

(b) Sketch $U(x)$, identifying the behavior at $x \to \pm\infty$, the value at $x = 0$, and the location and values of any local minima and maxima.
`[15 points]`

(c) Sketch the phase curves for $E = -\frac{2}{3}U_0$, $E = 0$, $E = \frac{2}{3}U_0$, and $E = 1.35\,U_0$. Identify which of the curves is a separatrix. Note that a given phase curve may have more than one disconnected component.
`[15 points]`

(d) Find an expression for the period of the bound orbit at $E = 0$, *i.e.* find $T(E = 0)$. Express $T(E = 0)$ fas a dimensionful quantity multiplied by a dimensionless integral.
`[15 points]`

[**2**] A forced, damped oscillator obeys the equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0 \cos(\omega_0 t) \quad .$$

You may assume the oscillator is underdamped. Note that the forcing frequency ω_0 is identical to the natural frequency of the unforced, undamped oscillator.

(a) Write down the most general solution of this differential equation.
`[20 points]`

(b) Your solution should involve two constants. Derive two equations relating these constants to the initial position $x(0)$ and the initial velocity $\dot{x}(0)$. *You do not have to solve these equations.*
`[15 points]`

(c) Suppose $\omega_0 = 5.0\,\mathrm{s}^{-1}$, $\beta = 4.0\,\mathrm{s}^{-1}$, and $f_0 = 8\,\mathrm{cm\,s}^{-2}$. Suppose further you are told that $x(0) = 0$ and $x(T) = 0$, where $T = \frac{\pi}{6}\,\mathrm{s}$. Derive an expression for the initial velocity $\dot{x}(0)$.
`[15 points]`