

PHYSICS 200B : CLASSICAL MECHANICS
HOMEWORK SET #1

- [1] Consider one-dimensional motion in the potential $V(x) = -V_0 \operatorname{sech}^2(x/a)$ with $V_0 > 0$.
- (a) Sketch the potential $V(x)$. Over what range of energies may action-angle variables be used?
- (b) Find the action J and the Hamiltonian $H(J)$.
- (c) Find the angle variable ϕ in terms of x and the energy E .
- (d) Find the solution for $x(t)$ by first solving for the motion of the action-angle variables.

Helpful mathematical identities :

$$\int_0^{\bar{u}(E)} du \sqrt{E + V_0 \operatorname{sech}^2 u} = \frac{\pi}{2} \left(\sqrt{V_0} - \sqrt{-E} \right) \quad \text{if } -V_0 < E < 0$$

$$\int du (E + V_0 \operatorname{sech}^2 u)^{-1/2} = \begin{cases} (-E)^{-1/2} \sin^{-1} \left(\sqrt{\frac{-E}{V_0+E}} \sinh u \right) & \text{if } -V_0 < E < 0 \\ E^{-1/2} \sinh^{-1} \left(\sqrt{\frac{E}{V_0+E}} \sinh u \right) & \text{if } E > 0 \end{cases}$$

where $\bar{u}(E) = \cosh^{-1} \sqrt{V_0/(-E)}$ in the first integral.

- [2] A particle of mass m moves in the potential $U(q) = A|q|$. The Hamiltonian is thus

$$H_0(q, p) = \frac{p^2}{2m} + A|q| \quad ,$$

where A is a constant.

- (a) List all independent conserved quantities.
- (b) Show that the action variable J is related to the energy E according to $J = \beta E^{3/2}/A$, where β is a constant, involving m . Find β .
- (c) Find $q = q(\phi, J)$ in terms of the action-angle variables.
- (d) Find $H_0(J)$ and the oscillation frequency $\nu_0(J)$.
- (e) The system is now perturbed by a quadratic potential, so that

$$H(q, p) = \frac{p^2}{2m} + A|q| + \epsilon B q^2 \quad ,$$

where ϵ is a small dimensionless parameter. Compute the shift $\Delta\nu$ to lowest nontrivial order in ϵ , in terms of ν_0 and constants.

[3] Consider the nonlinear oscillator described by the Hamiltonian

$$H(q, p) = \frac{p^2}{2m} + \frac{1}{2}kq^2 + \frac{1}{4}\epsilon a q^4 + \frac{1}{4}\epsilon b p^4 \quad ,$$

where ϵ is small.

- (a) Find the perturbed frequencies $\nu(J)$ to lowest nontrivial order in ϵ .
- (b) Find the perturbed frequencies $\nu(A)$ to lowest nontrivial order in ϵ , where A is the amplitude of the q motion.
- (c) Find the relationships $\phi = \phi(\phi_0, J_0)$ and $J = J(\phi_0, J_0)$ to lowest nontrivial order in ϵ .