PHYSICS 140B : STATISTICAL PHYSICS
HW ASSIGNMENT #3 SOLUTIONS

$(\mathbf{1})$ Consider a two-dimensional gas of fermions which obey the dispersion relation

$$\varepsilon(\mathbf{k}) = \varepsilon_0\Big((k_x^2 + k_y^2)\,a^2 + \tfrac{1}{2}(k_x^4 + k_y^4)\,a^4\Big) \ .$$

Sketch, on the same plot, the Fermi surfaces for $\varepsilon_{\rm F} = 0.1\,\varepsilon_0$, $\varepsilon_{\rm F} = \varepsilon_0$, and $\varepsilon_{\rm F} = 10\,\varepsilon_0$.

$(\mathbf{2})$ Using the Sommerfeld expansion, compute the heat capacity for a two-dimensional electron gas, to lowest nontrivial order in the temperature T.

$(\mathbf{3})$ ^{3}He atoms consist of an odd number of fermions (two electrons, two protons, and one neutron), and hence is itself a fermion. Consider a kilomole of ^{3}He atoms at standard temperature and pressure ($T = 293,\ {\rm K}$, $p = 1\,{\rm atm}$).

(a) What is the Fermi temperature of the gas? Assume $z \ll 1$ and justify this in part (b).

(b) Calculate $\mu/k_{\rm B}T$ and $z = \exp(\mu/k_{\rm B}T)$.

(c) Find the average occupancy $n(\varepsilon)$ of a single particle state with energy $\frac{3}{2}k_{\rm B}T$.

$(\mathbf{4})$ For ideal Fermi gases in $d = 1$, 2, and 3 dimensions, compute at $T = 0$ the average energy per particle E/N in terms of the Fermi energy $\varepsilon_{\rm F}$.

$(\mathbf{5})$ Obtain numerical estimates for the Fermi energy (in eV) and the Fermi temperature (in Kelvins) for the following systems:

(a) conduction electrons in silver, lead, and aluminum

(b) nucleons in a heavy nucleus, such as $^{200}{\rm Hg}$