PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #3

(1) The entropy for a peculiar thermodynamic system has the form

$$S(E,V,N) = Nk_{\rm B}\left\{\left(\frac{E}{N\varepsilon_0}\right)^{1/3} + \left(\frac{V}{Nv_0}\right)^{1/2}\right\},$$

where ε_0 and v_0 are constants with dimensions of energy and volume, respectively.

(a) Find the equation of state $p = p(T,V,N)$.

(b) Find the work done along an isotherm in the (V,p) plane between points A and B in terms of the temperature T, the number of particles N, and the pressures $p_{\rm A}$ and $p_{\rm B}$.

(c) Find $\mu(T,p)$.

(2) The Dieterici equation of state is

$$p\,(v-b) = RT\,e^{-a/vRT} \quad ,$$

with v the molar volume and with a and b constants.

(a) Wha are the dimensions of a and b?

(b) Find the coefficient of isobaric volume expansion, $\alpha_p = v^{-1}(\partial v/\partial T)_p$

(c) Find the conditions for the inversion temperature of throttling, $T\alpha_p = 1$ in terms of T and v.

(d) Define the temperature and pressure scales $RT_0 \equiv 2a/b$ and $p_0 \equiv 2a/b^2$. Define also the dimensionless temperature $\tau \equiv T/T_0$ and dimensionless pressure $\pi \equiv p/p_0$. Find and sketch the inversion curve $\pi(\tau)$.

(3) Consider the analog of the van der Waals equation of state for a gas if diatomic particles with *repulsive* long-ranged interactions,

$$p = \frac{RT}{v-b} + \frac{a}{v^2} \quad ,$$

where v is the molar volume.

(a) Find the molar energy $\varepsilon(T,v)$.

(b) Find the coefficient of volume expansion $\alpha_p = v^{-1}(\partial v/\partial T)_p$ as a function of v and T.

(c) Find the adiabatic equation of state in terms of v and T. If at temperature T_1 a volume $v_1 = 3b$ of particles undergoes reversible adiabatic expansion to a volume $v_2 = 5b$, what is the final temperature T_2?