PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #5

(1) Compute the density of states $D(E,V,N)$ for a three-dimensional gas of particles with Hamiltonian $\hat{H} = \sum_{i=1}^{N} A\,|\boldsymbol{p}_i|^4$, where A is a constant. Find the entropy $S(E,V,N)$, the Helmholtz free energy $F(T,V,N)$, and the chemical potential $\mu(T,p)$.

(2) For the system described in problem (1), compute the distribution of speeds $\bar{f}(v)$. Find the most probable speed, the mean speed, and the RMS speed.

(3) Consider a gas of classical spin-$\frac{3}{2}$ particles, with Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m} - \mu_0 H \sum_i S_i^z \ ,$$

where $S_i^z \in \big\{ -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2} \big\}$ and H is the external magnetic field. Find the Helmholtz free energy $F(T,V,H,N)$, the entropy $S(T,V,H,N)$, and the magnetic susceptibility $\chi(T,H,n)$, where $n = N/V$ is the number density.

$(\mathbf{4})$ Consider a system of identical but distinguishable particles, each of which has a non-degenerate ground state with $\varepsilon_0 = 0$, and a $g-$fold degenerate excited state with energy $\varepsilon > 0$. Study carefully problems #1 and #2 in the example problems for chapter 4 of the lecture notes, where this system is treated in the microcanonical and ordinary canonical ensembles. Here you are invited to work out the results for the grand canonical ensemble.

(a) Find the grand partition function $\Xi(T,z)$ and the grand potential $\Omega(T,z)$. Express your answers in terms of the temperature T and the fugacity $z = e^{\mu/k_{\rm B}T}$.

(b) Find the entropy $S(T,\mu)$.

(c) Find the number of particles, $N(T,\mu)$.

(d) Show how, in the thermodynamic limit, the entropy agrees with the results from the microcanonical and ordinary canonical ensembles.