Physics 152B
Spring 2024

Problem Set 4

Problem 1

Consider the tight binding energy band for a two-dimensional square lattice with lattice spacing a and nearest neighbor hopping t. Calculate the electrical conductivity from the Boltzmann equation within the relaxation time approximation for:

(a) the band nearly empty
(b) the band half-filled
(c) Writing the conductivities in the form given by the Drude formula in terms of a transport effective mass, what is the ratio of transport effective masses for the two cases?

Problem 2

Consider a body of volume V with simple cubic Bravais lattice structure , with conduction electrons in an energy band with energy versus k relation ε_k . Define

$$\frac{1}{m_k^*} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon_k}{\partial k^2}$$

(a) Assuming the body has zero electrical resistance, show that the current density $\vec{j}$ that develops when a magnetic field $\vec{H}$ is applied satisfies the London equation

$$\vec{\nabla} \times \vec{j} = -\frac{c}{4\pi\lambda_L^2}\vec{B}$$

and find an expression for the London penetration depth λ_L in terms of m_k^*'s.

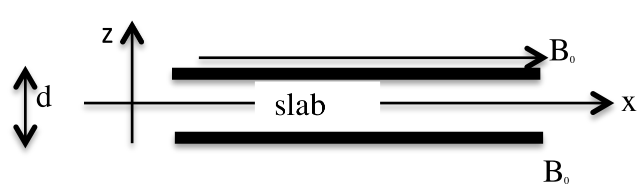
(b) Discuss the behavior of λ_L as function of the occupation of the band n and make a qualitative plot of λ_L versus n for $0 \le n \le 2$.

(c) For the two-dimensional square lattice described by a tight binding energy band with nearest neighbor hopping t=0.5eV, and lattice spacing a=2A, find the numerical value of λ_L in Angstroms.

Problem 3

Consider an infinite slab of superconducting material of thickness d in an applied magnetic field B_0 parallel to its surfaces. The slab is on the xy plane, its center is at z=0, the applied magnetic field is along the x direction. The density of superconducting electrons in this material is $n = 3.5 \times 10^{22}$ electrons/cm^3.

(a) Find an expression for the magnetic field inside the slab as function of position.
(b) What is the minimum thickness d (in cm) so that the magnetic field at the center of the slab is smaller than $B_0/100$? Assume the electron mass is the free electron mass.
(c) For B_0=300 Gauss (0.03T) find the speed of electrons v (in cm/s) at the surface and at the center of the slab for the thickness found in (b). Make a qualitative plot of v versus z. In which direction does the current flow?



Problem 4

Consider the Hamiltonian

$$H = -t\sum_{<ij>\sigma}(c^+_{i\sigma}c_{j\sigma} + h.c.) + U\sum_i n_{i\uparrow}n_{i\downarrow} + \sum_i H_i$$

with

$$H_i = \frac{1}{2M}P_i^2 + \frac{1}{2}Kq_i^2 + \alpha q_i(n_{i\uparrow} + n_{i\downarrow})$$

describing the interaction of electrons in a tight binding band with local oscillators with coordinate q_i and frequency $\omega = \sqrt{K/M}$.

(a) Denoting by |n> the oscillator ground state wavefunction at a site when there are n electrons at the site, find the overlap matrix element $S_{nn'} = <n|n'>$ in terms of α, ω and K.

(b) Assuming $\hbar\omega >> t$, find an effective Hamiltonian describing the motion of electrons in this band, of the form

$$H_{eff} = -t_{eff}\sum_{<ij>\sigma}(c^+_{i\sigma}c_{j\sigma} + h.c.) + U_{eff}\sum_i n_{i\uparrow}n_{i\downarrow}$$

and give expressions for t_{eff} and U_{eff} in terms of t, U, α, ω and K

(c) For what range of values of U will Cooper pairs in this system bind?

Problem 5

Lithium has electronic configuration $1s^2 2s^1$. It crystalizes in a bcc structure with lattice constant a=3.49A.

(a) Assume you have a crystal composed of Li^+ ions in the same crystal configuration. Estimate its magnetic susceptibility. Is it paramagnetic or diamagnetic?

(b) Same as (a) for a crystal of Li atoms, not ions.

Justify all steps.